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Radiation Limit for the Energy Gain of the p–^{11}B Reaction

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Abstract: The feasibility of positive energy yield in systems with the p–^{11}B reaction is considered here by considering refined (optimistic) data on the reaction rate. The analysis was carried out within the traditional framework for magnetic confinement systems, but without taking into account a particular type of plasma configuration. The energy balance was considered both for the ions and electrons. The balance of particles includes all species as well as the products of fusion (alpha particles). Calculations have shown that accounting for the content of thermalized reaction products (alpha particles) leads to an increase in radiation losses and a decrease in gain to $Q < 1$. In the steady-state scenario, the energy gain Q~5–10 can be obtained in p–^{11}B plasma, if only the fast (high-energy) population of fusion alpha particles is considered. For pulsed modes, the gain value is proportional to the content of alpha particles, and it is limited by the complete burn of one of the fuel components (boron), so it does not exceed unity. In the analysis we did not rely on any assumptions about the theoretically predicted mechanisms for increasing the cross section and the reaction rate, and only radiation losses (primarily bremsstrahlung) dramatically affect the gain Q. Thus, the regimes found can be considered as limiting in the framework of the classical concepts of processes in hot fusion plasma.

Keywords: p–^{11}B reaction; nuclear fusion; magnetic confinement; power balance; power gain

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1. Introduction

The aneutronic reaction p–^{11}B is attractive with the potential possibility of realization of a clean energy source based on thermonuclear fusion and therefore there is a high interest in finding possible concepts for its practical use for the production of electricity, other forms of energy, non-energy applications, and in the study of states of matter. Such studies have a rather long history [1–29].

The rate of the p–^{11}B reaction in plasma is relatively low even at very high temperatures $T > 100$ keV, and from power balance studies, it became clear that at such high temperatures, bremsstrahlung losses are practically equal to the energy released or greater [1–3,13,18,28]. In addition, for this reason, it is impossible to consider systems with a strong magnetic field in the plasma, since under such conditions the radiation losses will be even greater due to synchrotron radiation.

A noticeable yield of p–^{11}B fusion alpha particles was realized in experiments on the initiation of a reaction in the laser produced plasma [8]. In more recent works the alpha particle yield has been increased by many orders of magnitude going from 10^5 to more than 10^{10} alpha particles per laser shot [19,20,24,25]. The yield of alpha particles was detected in recent experiments in oscillating plasma with electrostatic confinement [22]. Additionally of note were experiments with p–^{11}B reaction in magnetically confinement plasma [30].

In [18] the search for possible regimes of p–^{11}B fusion is associated, in particular, with new data on the cross section of this reaction and the corresponding reaction rate [31]. In [31], it was shown that the reaction cross section is approximately 20% higher than the results presented in [32], which have recently been widely used. However, there are still large uncertainties in the measurements of the p–^{11}B fusion cross section. These uncertainties have stimulated new, and still ongoing, work on measurements of the cross section [29].

The present study was carried out within the traditional framework for magnetic confinement systems, but without taking into account a particular type of plasma configuration. An analysis of the energy balance for the plasma with the reaction p–^{11}B is of interest, considering the refined data on the reaction rate and accurate approximation for bremsstrahlung losses. The main aim of the present work was to find the conditions corresponding to the maximum efficiency characterized by fusion gain:

$$Q = W_{fus}/W_{in}, \tag{1}$$

where W_{fus} is the fusion energy, W_{in} is the energy input for heating and maintaining the plasma parameters (both W_{fus} and W_{in} are related to a certain time period).

It is noteworthy that for nuclear physics the reaction p–^{11}B is of essential interest, especially its mechanism [33–37]. The main channel can be represented as

$$p + {}^{11}B \rightarrow 3{}^{4}He + 8.68\ MeV. \tag{2}$$

There are also other reaction channels, but their contribution is small [33,34], therefore, from the point of view of thermonuclear fusion, we are not interested in them. Formally, transformation (2) proceeds in two stages:

$$p + {}^{11}B \rightarrow {}^{8}Be^{*} + {}^{4}He, \tag{3}$$

$${}^{8}Be^{*} \rightarrow {}^{4}He + {}^{4}He, \tag{4}$$

where the energy of an alpha particle in reaction (3) should be $E_{\alpha 1}$~4 MeV, the energy of each alpha particle in reaction (4) should be $E_{\alpha 2}$~2.3 MeV.

Since the decay of an excited nucleus ^{8}Be* occurs in a very short time (~ 10^{-16} s), then steps (3) and (4) should not be considered as independent. For this reason, in experiments, the spectrum of alpha particles has a maximum in the energy range of 3.5–5 MeV and a wide range at energies < 3.5 MeV [36]. Figure 1 shows the reaction scheme used for the calculations [35,36]. Figure 2 shows the calculated spectrum [35], which corresponds to the spectra obtained experimentally [37].

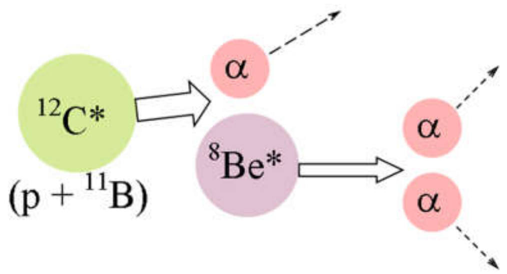


Figure 1. Schematic diagram of the reaction, where ^{12}C* is a compound nucleus (p + ^{11}B), α is alpha particle.

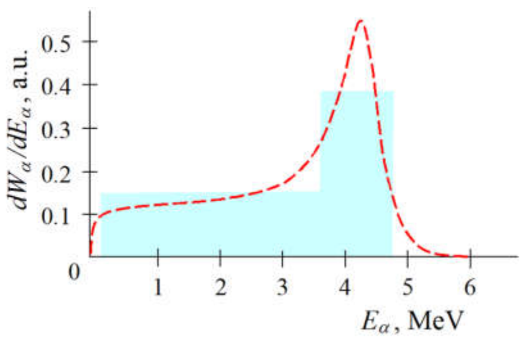


Figure 2. Energy spectrum [35] (dashed line) and schematic sketch of two characteristic energy ranges of alpha particles.

The energy spectrum of alpha particles is important for the energy balance of thermonuclear plasma since the fraction of energy transferred from alpha particles to the ion and electron components of the plasma depends on the energy of the alpha particle. A favorable regime can be realized if the alpha particles transfer almost all their energy to the ions. In this case, a high temperature of the ions is maintained, which is necessary for a high reaction rate. At the same time, the electron temperature is minimal, and, consequently, the radiation losses are minimal.

Even a slight increase in the reaction rate can essentially affect the improvement of the energy balance considered here. The "new" data on the p–^{11}B reaction cross section presented in [31] show higher values in the energy range of >500 keV compared to the "old" data [32]. In particular, at the incident proton energy E_p = 520 keV the "new" cross section is about 12% higher than the "old" one. The reaction rate is characterized by the fusion reactivity parameter $< \sigma v >$, i.e., the product of the reaction cross section and the relative velocity of the colliding particles, averaged over their distribution functions. Figure 3 shows a comparison of "new" and "old" data on the reactivity parameter for the case of Maxwellian velocity distributions of reacting ions with temperature T_i.

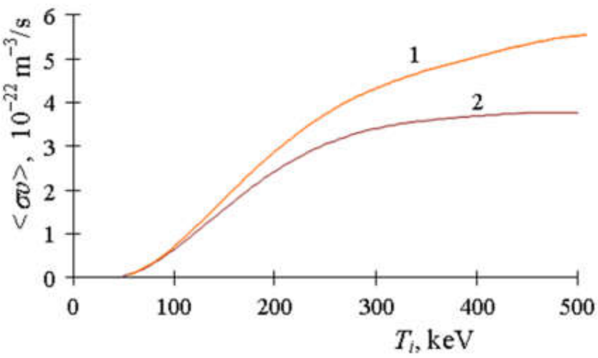


Figure 3. Comparison of "new" (1) and "old" (2) fusion reactivity values calculated according [31] and [32], respectively.

Note, in addition to the main reaction channel (2) the following reactions can occur in parallel:

$$\text{p} + {}^{11}\text{B} \rightarrow {}^{12}\text{C} + \gamma + 16.0\ \text{MeV}, \quad (5)$$

$$\text{p} + {}^{11}\text{B} \rightarrow {}^{11}\text{C} + n - 2.76\ \text{MeV}. \quad (6)$$

At relatively low energies, the cross sections of reactions (5) and (6) are much lower than the cross section for the main reaction (2). If the incident proton energy increases to E_p~4 MeV, the cross sections of reactions (2) and (6) become approximately equal in value. The reaction rates and product yields are determined by the reactivity parameter $< \sigma v >$, so it is the ratio of the reactivity parameters that determines the share of the yield realized in the corresponding reactions. Using the data [38], one can estimate that in the most important ion temperature range T_i = 200–500 keV, the ratios of the reactivity parameters of reactions (5) and (6) to the reactivity parameter of reaction (2) are ~10^{-4} and <$3{\cdot}10^{-3}$, respectively.

High-energy alpha particles produced in reaction (2) can interact with ^{11}B nuclei:

$$^{4}\text{He} + {}^{11}\text{B} \rightarrow {}^{14}\text{C} + \text{p} + 0.783\ \text{MeV}, \quad (7)$$

$$^{4}\text{He} + {}^{11}\text{B} \rightarrow {}^{14}\text{N} + n + 0.157\ \text{MeV}. \quad (8)$$

The cross sections of these secondary reactions become approximately equal to the cross section of the main reaction (2) at energies of the incident alpha particle E_α~3 MeV. At T_i~300 keV, the ratios of the reactivity parameters of reactions (7) and (8) to the reactivity parameter of reaction (2) are ~$5{\cdot}10^{-4}$ and ~$2{\cdot}10^{-2}$, respectively. In the case of a significant accumulation of alpha particles in the plasma, the yield of reaction products (8) is noticeable.

Note that in the presence of an admixture of the ^{10}B isotope in the fuel, in addition to the indicated parallel and secondary reactions, reactions with its participation can occur in the plasma, but we do not consider such reactions here. Taking into account both the reaction rates and the energy released in each of the reactions (2), (5)–(8), neutrons and radioactive products account for less than 1% of the energy yield. Therefore, the p–^{11}B fuel cycle is usually called aneutronic, although some insignificant level of radioactivity is not excluded.

The first estimates of p–^{11}B reactor parameters were made for inertial fusion systems. However, the required parameters turned out to be extremely hard both for systems with laser-driven targets [4] and for inertial-electrostatic confinement systems [5]. This situation remains typical even today, both for classical inertial fusion and for magneto-inertial systems. The physics of laser-plasma interactions [14,15] was used to analyze the possibilities of applying the reaction in inertial and magneto-inertial fusion schemes, including generation of pulses of an ultra-strong magnetic field by laser pulses [16]. We also note the idea of fusion in a system with oscillating fields in the interaction of positive boron ions with negative hydrogen ions [26].

Recently, the possibilities of systems with a plasma focus have been actively studied in application to p–^{11}B fusion [10–12]. The physics of the processes in the plasma focus [39,40] just makes it possible to provide such conditions when radiation losses do not lead to dramatic consequences, but, on the contrary, contribute to strong plasma compression in the focus (the so-called radiative collapse mode). In this case, of course, the question is how compression is limited by the development of constriction instability.

For preliminary estimates of plasma density n and confinement time τ, one can consider the value of the Lawson parameter $n\tau$~$6{\cdot}10^{21}$ m^{-3}s required for p–^{11}B fusion [41]. The relatively high Lawson parameter shows that a very long confinement time and high plasma density are required. For example, considering the magnetic confinement of plasma with a density n~10^{21} m^{-3}, one can find the required value of the magnetic field of ~ 10 T and higher. The corresponding confinement time will then be τ~10 s. The presence of a strong magnetic field in the plasma leads to very high losses due to synchrotron radiation. From this it followed that the fusion process must be organized in such a way as to increase the reaction rate, for example, due to the oncoming motion of components (protons and boron nuclei) with a higher relative velocity. Such concepts with beam-plasma fusion have been proposed in the projects of the CBFR (Colliding Beam Fusion Reactor) [6] and the ACT (Asymmetrical Centrifugal Trap) [9]. However, from the point of view of all processes included in the energy balance of such a non-equilibrium plasma, and especially taking into account relaxation [42], there are many questions on the feasibility of such approaches.

Note that there is a fundamental possibility of increasing the reaction rate when using spin polarized nuclei [43,44]. The possibility of applying this effect requires further research. Potentially, the cross section (and the rate) of p–^{11}B reaction can be increased by a factor of 1.6 compared to non-polarized nuclei.

The formation of an increased population of high-energy protons due to elastic nuclear interactions with fusion alpha particles was considered in [21,45]. In this case, the reaction rate should increase, but the influence of this effect should be considered correctly [21,46].

In our analysis, the energy balance was considered for both ion and electron components, as well as the balance of particles of all species, including fusion products (alpha particles). The content of products was estimated from the balance of their production in the reaction and the intensity of losses with a typical confinement time τ. The accumulation of products contaminates the plasma and leads to an increase in radiation losses due to impurities. In stationary plasma, the product content is so high that the gain is $Q < 1$. Probably, for pulsed regimes this problem is not like that for the steady-state scenario, but only under such conditions where the ion component is heated quickly. One can consider non-stationary regimes in which the pulse time τ_0 is less than the characteristic particle loss (confinement) time τ. In this case, essential accumulation of products can be avoided, and the plasma will remain relatively clean during the entire pulse.

Note, we did not rely on any assumptions about the efficiency of the theoretically predicted mechanisms to increase the cross section and the reaction rate. The considered modes are justified only by classical balance relations.

2. Methods

The balance of energy and particles in plasma is considered under the following simplifications. The intensity of particle losses, as well as energy losses associated with diffusion and heat conduction, are described by the characteristic confinement time τ. The plasma is considered spatially homogeneous. In this case, we do not associate the shape of the plasma with any particular geometry or any particular system. The equations describing the balance of fuel ions (protons and ^{11}B nuclei) are as follows:

$$\frac{dN_i}{dt} = -\frac{N_i}{\tau} - x_B N_p n_p < \sigma v > + S_i, \tag{9}$$

where N_i is the number of particles of a given type (i = p, ^{11}B), $n_i = N_i/V$ is the density (concentration) of particles, V is the plasma volume, $x_B = N_B/N_p$ is the relative content of boron ions, S_i is the intensity an external source of particles (optional, if it is required to maintain their specified content).

The number of alpha particles is found from the relation below:

$$\frac{dN_\alpha}{dt} = -\frac{N_\alpha}{\tau} + 3x_B N_p n_p < \sigma v > . \tag{10}$$

The number of electrons is determined from the quasi-neutrality condition below:

$$N_e = \sum_{i,\alpha} Z_i N_i, \tag{11}$$

where Z_i is the charge of the ion, the summation is carried out over all types of ions, i.e., protons, borons, and alpha particles.

The energy balance equations for fuel ions (i = p, ^{11}B) and electrons are considered in the following form:

$$\frac{1}{V}\frac{d}{dt}\left(\frac{3}{2}n_i k_B T_i V\right) = \alpha_i P_{fus} + P_{ext} - P_{i-e} - \frac{\frac{3}{2}n_i k_B T_i}{\tau}, \tag{12}$$

$$\frac{1}{V}\frac{d}{dt}\left(\frac{3}{2}n_e k_B T_e V\right) = \alpha_e P_{fus} + P_{i-e} - P_b - \frac{\frac{3}{2}n_e k_B T_e}{\tau}. \tag{13}$$

Here k_B is the Boltzmann constant; α_i and α_e are the fractions of the energy of charged products transferred to ions and electrons, respectively; P_{fus} is the power released in fusion reactions; P_{ext} is the external heating power (optional); $P_{i\text{–}e}$ is the power transferred from ions to electrons due to collisions; P_b is the bremsstrahlung power.

Fusion power is as follows:

$$P_{fus} = x_B n_p^2 < \sigma v > E_{fus}, \tag{14}$$

where E_{fus} = 8.68 MeV is the total energy of alpha particles, i.e., the fusion energy released in the reaction.

The power transferred from ions of each kind i to electrons in collisions is as follows:

$$P_{i-e} = \frac{\frac{3}{2}n_i k_B (T_i - T_e)}{\tau_{ie}}, \tag{15}$$

where τ_{ie} is the ion-electron collision time for ions of the considered type.

The collision frequency $\nu_{ie} = \tau_{ie}{}^{-1}$ decreases (τ_{ie} increases) with increasing electron temperature, and according to Equation (15), the difference between the ion and electron temperatures is greater for higher plasma temperatures. In a thermonuclear plasma with a temperature T_i~300 keV, the difference between the ion and electron temperatures can reach ~100 keV. It is known that relativistic effects become noticeable in the process of electron–ion energy exchange at electron temperatures T_e > 100 keV [47]. In particular, according to [2], considering the relativism of electrons, in the range T_e = 100–200 keV, the collision frequency $\nu_{ie} = {\tau_{ie}}^{-1}$ is 9–13% higher than the classical non-relativistic values. This effect was taken into account when calculating (15).

The values α_i and α_e included in the energy balance Equations (12) and (13) and the contribution of fast particles to the reaction rate are calculated on the basis of the velocity distribution function. In the distribution of alpha particles, one can conditionally distinguish between "thermal" and "fast" populations. The thermal population is characterized by a distribution close to the Maxwellian and energies of the order of $k_B T_i$. The energy of fast alpha particles E has a value in the range $k_B T_i << E < E_\alpha$, where E_α is the energy of the alpha particle birth. The total number of alpha particles is determined by Equation (10). To estimate the number of fast alpha particles, one can use approximate expressions for the distribution function of fast particles [48] for the case when the characteristic Coulomb slow-down time τ_s is large compared to the confinement time, i.e., $\tau_s >> \tau$. For a group of particles produced with a velocity $v_{\alpha 0} = \sqrt{2E_\alpha/m_\alpha}$ (m_α is the mass of an alpha particle), such a velocity distribution function in the region of superthermal energies has the form [48] below:

$$f_\alpha(v) \approx \frac{\dot{N}_\alpha \tau_s}{4\pi(v^3 + v_c^3)}, \tag{16}$$

where $\dot{N}_\alpha = 3x_B N_p n_p < \sigma v >$ is the number of alpha particles produced per unit time as a result of the reaction; τ_s is the slow-down time; v_c is the critical velocity (velocity at which slow-down on electrons is equal to slow-down on ions).

The corresponding number of fast alpha particles is as follows:

$$N_\alpha \approx \frac{1}{3}\dot{N}_\alpha \tau_s \ln\left[(E_\alpha/E_c)^{3/2} + 1\right], \tag{17}$$

where $E_c = m_\alpha v_c^2/2$ is the critical energy.

The fraction of energy transferred by an alpha particle to electrons is as below [49]:

$$\alpha_e \approx \frac{E_c}{E_\alpha}\int_0^{E_\alpha/E_c} \frac{x^{3/2}}{1 + x^{3/2}}dx. \tag{18}$$

The velocity distribution function is represented by Equation (16) corresponding to the isotropic plasma (limiting case in a certain sense). Such an approximation can be used in the case when the features of the plasma configuration are not considered. We emphasize that (16) describes only the high-energy population of alpha particles at $E \gtrsim E_c$. Outside this energy range, the relaxing alpha particles form a thermal population with a temperature close to the fuel ion temperature. The critical energy depends on the electron temperature. At T_e~150 keV, the critical energy is E_c~1 MeV; therefore, when analyzing the influence of fast alpha particles, we do not consider particles with lower energies. Since Equations (17) and (18) are based on the velocity distribution function (16), by using these expressions approximate estimates can be obtained. The energy spectrum of the produced alpha particles (Figure 2) depends on the energy, so some averaging of (17) and (18) over the energy is necessary. For accurate calculations, it is necessary to have an exact expression for the spectrum or its high-precision fit, which cannot be extracted with high accuracy from published experimental data. It also makes no sense to carry out quantum mechanical calculations due to the approximate nature of Equations (16)–(18). Therefore, we use a rather rough algebraic approximation, in which we take into account the features of two energy ranges of the spectrum of born alpha particles with only one value of the characteristic energy for each range. For the high energy range (>3.5 MeV), we take the following parameters: characteristic energy E_1 = 4.5 MeV, weight factor g_1 = 0.33. For the range of relatively low energies (1–3.5 MeV), we take the following: E_2 = 2.09 MeV, g_2 = 0.67. The effect of spectral features outside the indicated ranges is not very important, since they account for about 11% of the born alpha particles [35]. Note that such choice qualitatively reflects the features of the spectrum and corresponds to the total energy of the alpha particles. The averaging operation in this case has the simplest form: $\langle\varphi(E_\alpha)\rangle \approx \sum_{k=1,2} g_k\varphi(E_k)$, where $\varphi(E_\alpha)$ means the averaged energy dependence. Note that alpha particles produced with energies > 3.5 MeV make the largest contribution to the total content of fast particles. They also transfer a noticeable proportion of their initial energy to electrons. Alpha particles, born with lower energies, give almost all of their energy to plasma ions. The estimates made showed that, on average, the fraction of alpha particle energy transferred to electrons is α_e~0.05.

Bremsstrahlung occurs when electrons collide with ions and electrons. Such radiation is not absorbed by the plasma of thermonuclear parameters and is not reflected from the reactor walls surrounding the plasma. Therefore, just like neutrons, this is an inevitable channel of energy loss from plasma. Considering the content of the reaction products confined in the plasma, bremsstrahlung can exceed the heating of the plasma by the products of the p–^{11}B reaction. Therefore, the energy loss due to bremsstrahlung must be calculated with the highest possible accuracy. The results of numerical calculations of bremsstrahlung in electron–ion and electron–electron interactions and rigorous analysis of the approximating formulas for the bremsstrahlung power are analyzed in detail in [50] for a wide range of electron temperatures (from low to ultra-relativistic values). In this work, we use the method given in [50] for calculating bremsstrahlung losses. The structure of the formula for bremsstrahlung power is as follows:

$$P_b = C_b n_e^2 (Z_{eff}^2 \varphi_i(T_e) + \varphi_e(T_e)), \tag{19}$$

where C_b is some constant, $Z_{eff}^2 = \sum_i Z_i^2 n_i/n_e$ is the effective square of the ion charge, $\varphi_i(T_e)$ and $\varphi_e(T_e)$ are functions of the electron temperature that take into account electron–ion and electron–electron bremsstrahlung, respectively.

In a non-stationary mode with a working pulse duration of $\tau_{0,}$ the gain Q is determined directly by Formula (1), where

$$W_{fus} = \int_0^{\tau_0} P_{fus} V dt, \tag{20}$$

$$W_{in} = \left(\sum \frac{3}{2} n_{i0} k_B T_{i0} + \frac{3}{2} n_{e0} k_B T_{e0}\right) V_0 + \int_0^{\tau_0} P_{ext} V dt, \tag{21}$$

the symbol "0" marks the initial parameters, i.e., the starting plasma parameters.

In stationary mode, $d(\ldots)/dt = 0$, i.e., the left parts of Equations (9), (10), (12), and (13) are equal to zero. Energy losses must be compensated by heating by fusion the alpha particles as well as heating from an external source. In accordance with (12), (13), the absorbed power of external heating is as follows:

$$P_{ext} = P_b + \frac{\frac{3}{2} n_i k_B T_i + \frac{3}{2} n_e k_B T_e}{\tau} - P_{fus}, \tag{22}$$

and the gain is then the following:

$$Q = \frac{P_{fus}}{P_{ext}} = \frac{1}{\frac{P_b}{P_{fus}} + \frac{\frac{3}{2}(n_i/n_p) k_B T_i + \frac{3}{2}(n_e/n_p) k_B T_e}{n_p \tau (P_{fus}/n_p^2)} - 1}. \tag{23}$$

It can be seen from (14) and (19) that in a plasma with a certain composition of components, the ratio P_b/P_{fus} depends only on the temperatures T_i and T_e, while the value (P_{fus}/n_p^2) depends only on T_i. Thus, Equation (23) connects the following quantities: Q, T_i, T_e, and the product $n_p\tau$. But the temperatures T_i and T_e are not independent since they are interconnected by the energy exchange between the ion and electron components.

As we already noted, the accumulation of reaction products in the plasma is a problem for p–^{11}B fusion. From Equation (10) it is easy to obtain an estimate of the content of products in the stationary mode as below:

$$N_\alpha = 3x_B N_p n_p < \sigma v > \tau, \tag{24}$$

$$x_\alpha = N_\alpha^S/N_p = 3x_B < \sigma v > n_p\tau, \tag{25}$$

where for the stationary regime the number of protons and boron nuclei, as well as their density, are assumed to be constant.

The content of products can be relatively small, but their presence in plasma leads to dramatic changes in the value of Q.

From Equation (23) it can be seen that in the stationary mode, the following conditions correspond to the highest gain: (i) as long as possible confinement time; (ii) minimal radiation losses, i.e., pure plasma, practically free of fusion products and other impurities. For a stationary scenario, these conditions cannot be met simultaneously. Nevertheless, approaching these ideal conditions allows one to estimate the theoretical limit for the gain. Equation (23) shows that if

$$P_{fus} \geq P_b, \tag{26}$$

then a self-sustaining reaction (without external heating, $Q \to \infty$) is possible.

High gain Q imposes a slightly softer requirement, namely $P_{fus} \approx P_b$. In any case, it follows from these conditions that the content of fusion products (alpha particles) in the plasma must be minimal, otherwise bremsstrahlung losses will be unacceptably high. We can say that the gain is restricted by the radiation limit.

Let us consider non-stationary (pulsed) regimes. When the pulse duration $\tau_0 << \tau$, the relative content of alpha particles is limited in growth by the value below:

$$x_\alpha = N_\alpha/N_p = 3x_B n_p < \sigma v > \tau_0. \tag{27}$$

At the same time, according to (20) and (21), at constant temperature and density

$$Q = \frac{W_{fus}}{W_{in}} = \frac{x_B n_p^2 < \sigma v > E_{fus}\tau_0}{\sum \frac{3}{2} n_i k_B T_i + \frac{3}{2} n_e k_B T_e}. \tag{28}$$

Using (27) we obtain the following:

$$Q = \frac{2}{9} \frac{x_\alpha}{1 + x_B + (1 + 5x_B)\frac{T_e}{T_i}} \frac{E_{fus}}{k_B T_i}. \tag{29}$$

As can be seen from the resulting expression, power gain is higher, if the content of products (alpha particles) is higher. However, at the same time radiation losses cannot exceed a certain value. This leads to a restriction on x_α and, respectively, on Q.

Let us consider an approach to estimating the maximum achievable Q for a pulsed regime. This value corresponds to the conditions that one of the fuel components (in this case it is boron-11) burns out completely. In this case, the released fusion energy is proportional to the fusion energy, and the supplied energy corresponds to the characteristic temperature. These considerations lead to the following expression for the limiting gain:

$$Q = \frac{2}{3} \frac{x_B}{1 + x_B + (1 + 5x_B)\frac{T_e}{T_i}} \frac{E_{fus}}{k_B T_i}. \tag{30}$$

To estimate this value, we take T_i~300 keV, T_e~150 keV, x_B~0.2. Then we find that, according to Equation (30), the maximum gain will be Q~1. The reason for such a low value of Q is the very large requirement to heat the fuel in order to achieve the necessary thermonuclear temperatures. In the next section, we consider the limiting parameters of proton-boron plasma with a stationary fuel composition.

3. Results

Note that the purpose of our analysis is to find a "window of parameters" in which one can expect high gain, at least under somewhat idealized conditions. The calculations showed that the results are highly sensitive to small variations in the parameters of the model. In particular, an increase in the reaction rate by ~10% makes it possible to find conditions with a maximum gain not with Q~1, but with Q~10. As we noted above, the values of the cross section obtained in [31] are slightly higher than the values in [32], which up to that time looked the most optimistic. Therefore, we provide a comparison for two cases: (i) the reaction rate corresponding to the "old" data [32], and (ii) the reaction rate corresponding to the "new" data [18,31].

In the first series of calculations, we considered clean plasma, i.e., the content of alpha particles was not taken into account. This approach is similar to the assumption used in [18], and our results are also close to the results of that work. In the calculations, we were guided by the value of the Lawson parameter $n\tau$~$6{\cdot}10^{21}$ m^{-3}s estimated in [41]. Figure 4 shows the gain Q and the ratio of the fusion power to the bremsstrahlung loss power P_{fus}/P_b as functions of the ion temperature. The electron temperature determined from the balance Equations (12) and (13) for the steady-state regime is also shown. As one can see, the use of "new" data for the reaction rate led to the changing in the theoretical limit of the value of P_{fus}/P_b upward from ~0.8 to >1, and accordingly the opening of the "ignition window".

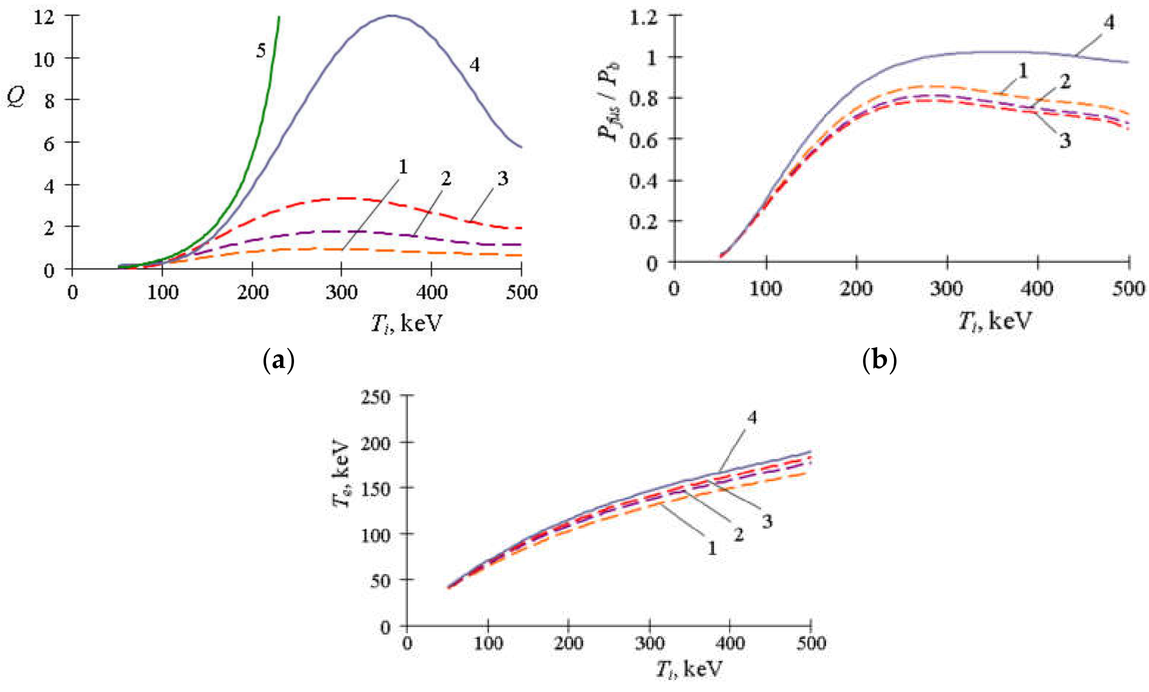


Figure 4. (**a**) Fusion gain, (**b**) fusion to bremsstrahlung power ratio, and (**c**) electron temperature versus ion temperature for the case of clean plasma (the content of alpha particles is not taken into account) for "old" (dashed) and "new" (solid) data on the reactivity. 1—$n_p\tau = 1.7{\cdot}10^{21}$ m^{-3}s, $x_B = 0.19$; 2—$n_p\tau = 5.0{\cdot}10^{21}$ m^{-3}s, $x_B = 0.19$; 3—$\tau \to \infty$, $x_B = 0.19$; 4—$n_p\tau = 16.2{\cdot}10^{21}$ m^{-3}s, $x_B = 0.14$; 5—$\tau \to \infty$, $x_B = 0.14$. Data for case 5 are not shown in panels (**b**,**c**) as they very close to case 4 at $T_i < 220$ keV, and there is no energy balance at higher temperatures for these ideal conditions.

Note that balance Equations (12) and (13) retain the similarity in the parameter $n_p\tau$, where n_p is the density (concentration) of protons. Therefore, the results presented in Figure 4 and below are characterized not by the value of the required confinement time τ, but by the complex double product parameter $n_p\tau$. The fuel composition (value x_B) for optimal conditions is somewhat different when using the "old" and "new" reaction rates. Figure 4 shows the data for boron content x_B, which characterizes the maximum gain.

In the second series of calculations, we assumed that the confinement time of alpha particles is determined by a finite value τ, so this time should not be too short or too long. The calculation results are shown in Figure 5. As the analysis showed, the limiting gain does not exceed unity. The content of alpha particles x_α in these calculations was determined by Equation (25). If the confinement time τ is too short, the content of alpha particles is relatively small, but the plasma losses are large. Modes with $P_{fus}/P_b > 1$ are possible, but at the same time Q is low due to plasma losses. With a long confinement time τ, the content of alpha particles is high and, accordingly, the losses due to bremsstrahlung are high. Fast alpha particle content x_α is somehow lower in comparison with the value given by Equation (25).

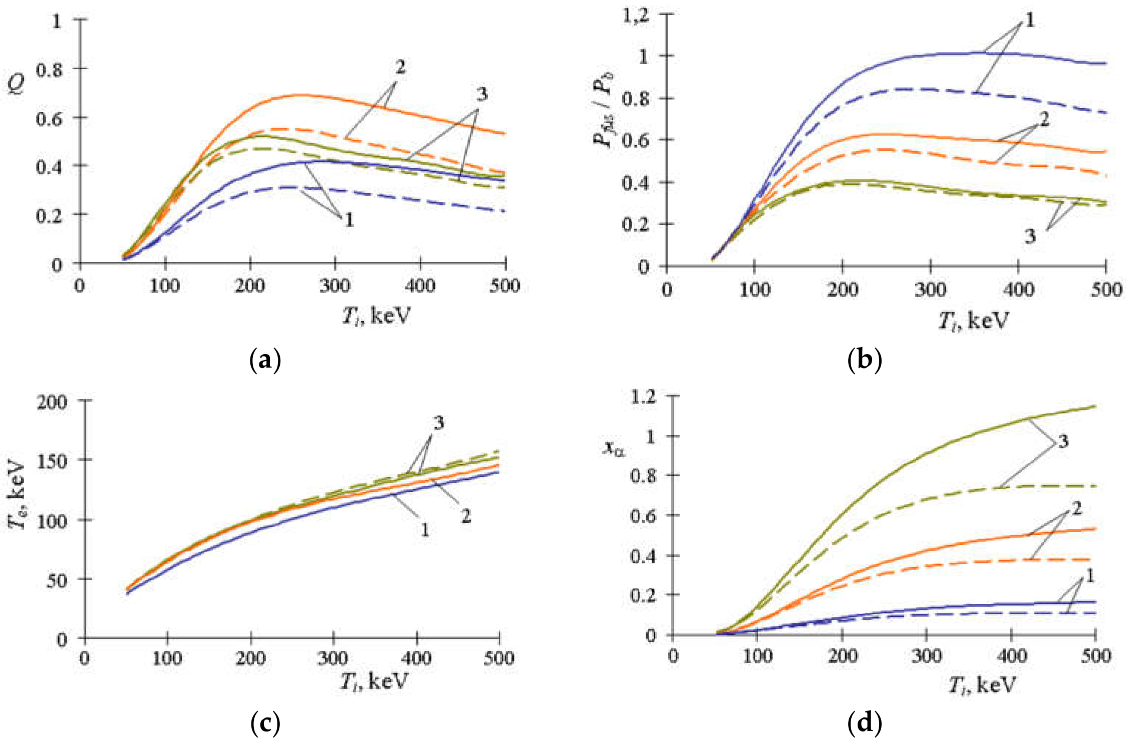
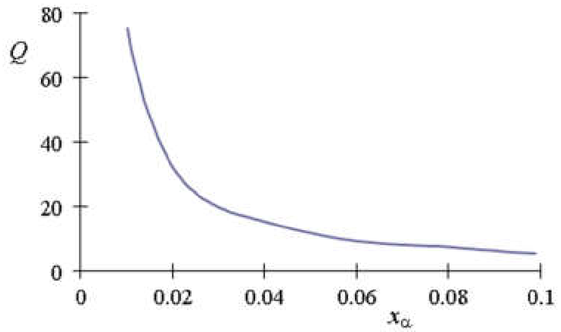
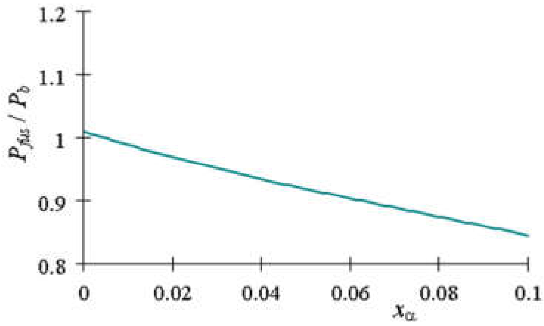


Figure 5. (**a**) Fusion gain, (**b**) fusion to bremsstrahlung power ratio, (**c**) electron temperature, and (**d**) fusion alpha particle content versus ion temperature under stationary conditions with x_B = 0.19 for "old" (dashed) and "new" (solid) data on the reactivity. 1—$n_p\tau$ = 0.5·10^{21} m^{-3}s; 2—$n_p\tau$ = 1.7·10^{21} m^{-3}s; 3—$n_p\tau$ = 3.5·10^{21} m^{-3}s.

If the accumulation of alpha particles is completely neglected, the most optimistic regimes correspond to an infinitely long confinement, i.e., $\tau \to \infty$ (for fuel ions and electrons). Within the framework of this assumption, one can analyze the influence of the content of alpha particles in the plasma, considering this value as a given parameter. Figure 6 shows the gain and ratio P_{fus}/P_b versus the given alpha particle content.



(a)



(b)

Figure 6. (**a**) Fusion gain, and (**b**) fusion to bremsstrahlung power ratio versus the given alpha particle content for "new" data on the reactivity at T_i = 375 keV, x_B = 0.14, confinement time for fuel ions and electrons $\tau \to \infty$.

It probably makes no sense to consider the complete removal of alpha particles within the framework of the thermal scheme, since it is hardly possible to implement such regimes. Therefore, consider an idealized scenario when high-energy alpha particles transfer their energy to ions (protons and borons) and plasma electrons, slow down to thermal energies, and then they are removed from the plasma. The number of alpha particles in the high-energy (superthermal) range depends on the intensity of the reaction and the temperature of the electrons. In this case, the total number of alpha particles corresponds only to such a high-energy population. Within the framework of the described idealized scheme, we can consider a hypothetical case when the confinement time for fuel ions and electrons is $\tau \to \infty$, but the confinement time for thermalized alpha particles is $\tau \to 0$. For such conditions, the content of fast alpha particles $x_\alpha = N_\alpha/N_p = n_\alpha/n_p$ (here N_α and n_α are the number of fast particles and their density, respectively) is shown in Figure 7.

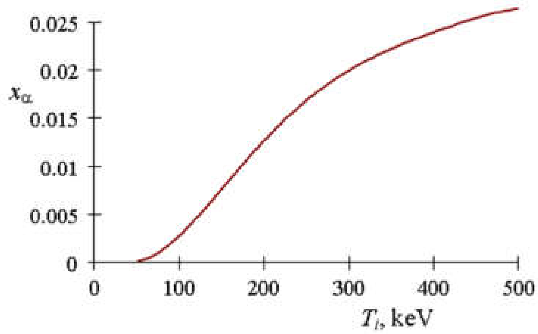


Figure 7. The content of fast alpha particles versus ion temperature for "new" data on the reactivity at x_B = 0.14, $\tau \to \infty$.

As can be seen in Figure 7, the content of fast alpha particles is about 2.5 times lower than the content, at which Q~10 can be expected. Therefore, further searches for optimistic regimes can be apparently associated with the study of methods for forced removing of thermalized alpha particles from the plasma core (a kind of "pumping out"). The physical principles of such "pumping out" have been theoretically developed [51,52], but have not yet been tested in experiments.

4. Discussion and Conclusions

The use of both updated ("new") cross section and reaction rate showed the possibility of finding optimistic regimes for p–^{11}B fusion. In particular, a parameter window is possible in which the ratio of fusion power to bremsstrahlung power is $P_{fus}/P_b > 1$. However, the existence of such a window turned out to be very sensitive to the features of the model and its characteristic parameters. Unfortunately, within the framework of the classical concept of plasma with Coulomb collisions, gain of $Q > 5$ can be obtained only if the thermalized alpha particles (the reaction products) are removed from the plasma.

At the present time, the development of schemes with the removal of alpha particles is the most realistic way towards the implementation of p–^{11}B fusion energy. Non-equilibrium and non-stationary systems, of course, should also be considered in order to understand their physical features and the real possibilities for improving the energy balance compared to the classical case.

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