Proceeding Paper

Notes on the Cross-Level Game †

Yu Chen

Department of Mathematics, Renmin University of China, Beijing 100038, China; ychen@ruc.edu.cn
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Abstract: Up till now, the research on game theory has concentrated on the game issues between agents, which are homogeneous and at the same level in the complex hierarchical system. There have already been many fruitful results that have been successfully used in many fields, such as economics, public management, sociology, and so on. However, in real life, the more frequently faced game problems are cross-level games, which are among agents from different levels. For example, in a supply chain, single factory and whole chain; in the public management, the single family and the whole community; in a tropical rain forest, single species and whole eco-system; in biology, an animal and the bacterial group in its body; and so on. Up till now, similar topics had not been discussed in detail. This situation seriously limited the research and applications for game theory. In this paper, the author introduces their own ideas for the difficulties and possible approaches on this issue. Some suggestions have been provided at the end. The main suggestion of this paper is to learn from the results of the complexity study, to focus on the payoff function, and to find an approach for the cross-level game model.

Keywords: game theory; complexity study; hierarchical structure

1. Game Theory—What's Fun and Difficult Points

Classically speaking, game theory became an academic field starting with the work of von Neumann [1,2]. After being supplemented and improved by Nash and other scholars, it quickly became a unique, interesting and distinctive subject field; this historical process has been summarized by many excellent textbooks [3,4] which need not be repeated. For a time, game theory research was very popular. The attribution of many Nobel prizes is a strong proof of this enthusiasm.

The appeal of game theory lies in its innovative ideas and its wide application in many fields. The cool analysis of the game process and the unexpected quantification of the treatment make people feel enlightened and refreshed. Everyone who has just entered this field will be deeply attracted and shocked by it. For people accustomed to the traditional so-called scientific thinking, this is a very new way of thinking. The absolutist thinking tradition and the shackles of monism make people accustomed to the absolutist right and wrong and unconsciously limited to the linear thinking pattern. It is for this reason that the emergence and development of game theory has opened people's eyes and enlightened them. More importantly, the rapid application of new ways of thinking in game theory within so many fields, especially economics, sociology, management and politics, raised great hopes that game theory could bring about fundamental change. This was the background and origin of the game theory craze of the second half of the last century.

But therein lies the problem. The unavoidable fact is that the game theory craze waned with the turn of the century, as many colleagues have pointed out. Where is the problem? Opinions on this have varied, and many people have proposed various explanations and answers. Here, the author boldly puts forward an explanation: the reason is that up till now, game theory research had not completely gotten rid of the simplification of traditional thinking; absolute thinking has shackled and limited the study of the game problem

between homogeneous subjects, or, to put it another way, there is still an underappreciation of the complexity and importance of systems, instead people are trying to simplify and quantify complex realities. This greatly limits the development and application of game theory as it excludes the rich and numerous realistic issues from the field of view of game theory research. One of the most prominent problems is the neglection of research on the game between agents of different qualities across levels. Based on this consideration, the author believes that it may be a breakthrough for the study of game theory to attach importance to the experience in the field of complexity research, take the game problems between different agents across levels as a breakthrough point, and explore new game theory methods. This is the main proposal of this paper.

2. Introduction to Complexity Research and Complex Adaptive Systems (CAS)

The idea proposed by the author is based on the theory of complexity study and complex adaptive systems. To illustrate the direction suggested by the authors, these two concepts are briefly introduced in this section, and readers who are already familiar with them may skip this section.

Complexity Study is a new research idea that emerged in the second half of the last century. One of its main exponents was Herbert Simon. In view of the absolutization and the simplification of traditional thinking, he put forward the concept of valuing complexity, diversity and hierarchy, as well as a research method based on practice, and carried out creative research and an investigation on the complex system in many fields, which has had a huge impact on the scientific community. His main representative works include *Artificial Science* [5] and *Practice-based Microeconomics* [6]. In particular, he proposed the concept of a semi-decomposable hierarchical structure which is very inspiring for the improvement of game theory.

As for the related research of a complex adaptive system (CAS), it is predominantly John Holland's explanation and concretization of Simon's idea. In his many books, he has convincingly and vividly explained the formation and evolution processes of complex systems with a large number of cases from different scientific fields, and conducted creative research on complexity and complex systems. For example, there are already Chinese translations of *Hidden Order* [7] and *Emergence* [8], but there are still no Chinese translations of *Complexity* [9] and *Signals and Boundaries* [10]. In these works, Holland puts forward a relatively complete concept and theory on the formation and evolution of complex adaptive systems. It is precisely these novel results that inspire us to develop and improve game theory.

This paper has notes on the study of these two scholars' works, and the theme that haunts me throughout is the transformation of game theory. Here are some thoughts from this learning process.

3. What Mind-Sets Need to Change

It has been pointed out above that the transformation of game theory needs to change the traditional mode of thinking. Then, to be specific, what outdated and traditional thinking patterns are restricting and hindering us? In my opinion, there are four main points.

Firstly, the pursuit of an absolute and ultimate theory. Shortly before his death, the famous scholar Stephen Hawking said "Until now, most people have implicitly assumed that there is an ultimate theory and that we will eventually discover it. In fact, I myself have said that we will find this theory soon. But M-theory makes me wonder if this is true. Perhaps it is impossible to formulate an ultimate theory of the universe in a finite number of propositions", Ref. [11]. At the end of the article, he emphasized quite clearly: "Some people will be very disappointed if there is no ultimate theory that can be derived from the finite number principle. I used to be in that camp. But I have changed my mind. Now I am glad that our quest for knowledge never reaches its end. There is always the challenge of making new discoveries. Without that challenge, we would stagnate. Godel's theorem ensures that mathematicians always have something to do, and I think M-theory will do

the same for physicists". This quotation is a bit long, but it's important. Hawking sharply pointed out an important watershed between modern science and recent science, that is, the suspicion of an absolute ultimate truth starting from Godel. Game theory textbooks to date, however, still present us with this set of thinking. Taking a look at our current game theory textbooks, it is clear that the reality is not optimistic, and it is no accident that game theory research is at a low ebb today.

Secondly, traditional game theory studies only consider the difference and comparison of quantitative indicators, and completely ignore the qualitative difference, especially the qualitative difference between players in the game. The research is limited to the individuals who are completely homogeneous and have the same benefit function, which greatly simplifies and deviates from the reality. It can be seen that most of the game projects involved in reality tend to be a conflict of interest between heterogeneous agents, and the so-called intra-species struggle between the so-called completely equivalent agents is only a few extremely simple cases.

Thirdly, only the static states are examined, ignoring the process of evolution. A static, fixed way of thinking greatly limits the research view of game theory. Many cases remind us that most games are a process in which the participants constantly change their behavior patterns and even physical structures. The fixed behavior pattern, fixed value judgment and fixed physical structure mean that the study of game theory inevitably encounters developmental obstacles. In fact, the issues of time, evolution, and development process hardly have any place in the traditional game theory research, or are even rarely involved in it.

Fourthly, there is a simplified understanding of the word "level". Although the word "level" has existed since ancient times and is often mentioned in traditional disciplines, it usually has only quantitative meaning, that is, the difference between the upper and lower levels is only quantitative without a qualitative difference. For example, the agents at the upper level are often regarded as a simple combination or accumulation of agents at the lower level. The attributes of the upper subject are simply the sum or accumulation of the corresponding attributes of the lower subject. For Aristotle's maxim, "the whole is greater than the sum of its parts", is often ignored by people. With this understanding, levels are simply understood as the accumulation of quantities, completely ignoring the emergence of new qualities. Because of this, the world is often inappropriately simplified and banal, and the colorful objective world has lost its vitality.

Although these situations are not unique to the research field of game theory, many other research fields also exist; more or less, however, the undeniable fact is that in the field of game theory, this situation is particularly prominent, the impact is particularly significant, and it is worthy of a lot of attention from researchers.

4. How to Start Thinking—Key Topics to Explore

So where to start? As a starting point, we believe that we can concentrate on the following key issues as breakthrough points to the dilemma of game theory research.

Firstly, the individual division of labor and differentiation. Focusing on games between heterogeneous players is a critical first step. The most typical example of this is the emergence of a division of labor. Whether it is for the consideration of improving efficiency or under the objective pressure of individual differences, once the division of labor occurs, individual differentiation will inevitably lead to the difference of interest orientation, the so-called difference of benefit function will appear, and the assumption that the benefit function is the same will be broken. Take Adam Smith's famous division of labor in needle-making, for example. Workers in different processes will have different demands on their share of the benefits, or their understanding of the fairness of the distribution. Naturally, these distinctions or differences need to be analyzed and studied in detail. There are many phenomena and laws worth further study here.

Secondly, emergence, that is, the generation of new levels and subjects. If the phenomenon described in the previous point is the differentiation and evolution of subjects

at the same level, then the more complex phenomenon that follows is the emergence of a new subject: the whole production line. The interest orientation of this subject is different from that of each individual worker, and new levels and subjects emerge. This detailed process deserves our further careful analysis and study. Obviously, this is by no means a simple sum, nor can it be summarized by the traditional benefit estimation in the game theory model. We need new ideas, new methods, and new tools.

Thirdly, the relationship between the overall and individual benefits. In the words of a common saying, it is the relationship between the big self and the small self. The relationship between the state and the family, the relationship between enterprises and employees, the relationship between the local and the state, and the relationship between globalization and the national interests of countries are all these specific cases of this issue. Are there any universal phenomena or laws? This is well worth investigating.

Fourthly, the phenomenon of "weird circle" and "intergenerational relatives". When we study more cross-level game problems, more and more rich phenomena appear. Just two examples are given here. One is the so-called "weird circle". Douglas R. Hofstadter, the author of the famous "GEB—an eternal golden belt" [12], introduced the so-called winding hierarchy—the "weird circle" in his new book *I am a Strange Circle* [13], which further breaks the one-way concept of hierarchy and leads us from the linear and one-way concept of a pagoda-shaped hierarchy to a more colorful and bizarre concept of hierarchy. Readers who are interested may wish to read and enjoy the infinite scenery across the levels. Another interesting phenomenon is the intergenerational relationship. Simply put, grandsons and grandfathers unite against the middle generation, or "The mantis stalks the cicada, unaware of the oriole behind". The combination of intergenerational interests has manifested itself in many social and political fields. All this demonstrates the richness and quirkiness of cross-level, heterogeneous agent game problems, and isn't this a very worthy area of research?

5. Suggestions for Further Research—Potential Application Areas as Entry Points for Research

As mentioned earlier, our research needs to start from practice, not from ideas, as Simon said in practice-based microeconomics. To be more specific, do not start from the concept, but from a number of typical complex systems, then investigate and analyze a number of realistic fields and cross-level game problems to understand the new ideas and to explore the germination of new game theory methods.

Based on what we have read about the work of Simon, Holland, and others, as well as our own practice, the following areas of research are worthy of serious attention.

The first, of course, is the economic system. From the previous introduction, the reader has seen that the complexity of the economic system is undoubtedly the most important source for the formulation of the game problem across levels. The debate between macroeconomics and microeconomics is undoubtedly one of the most typical hierarchical debates. The so-called dispute between virtual economy and real economy, the mystery of globalization, supply chain management (which is not a typical cross-level) are all a different nature of the subject of the game problem. Therefore, as an important development direction of information economics, as well as the main application field of game theory across levels, the continuous attention and investigation of these game problems is undoubtedly the most important source of inspiration for researchers. We are looking forward to it!

The second is social management and public management. This is also an important area where many cross-level game problems have been studied. The emergence of the police system in small towns in western United States, the famous tragedy of the Commons, and the gains and losses of representative democracy are all related to this. The rich practical cases have provided and will continue to provide rich inspiration for the investigation and research of cross-level game problems.

The third area is biology and environmental science. In Holland's work, tropical jungle ecosystems are often cited as examples of complex adaptive systems. Whether it is a specific ecological environment from biological populations in a specific area, such as grassland degradation, forest fires to oceans and lakes, or global warming, these are all game issue for heterogeneous subjects. Moreover, this is a hot topic that everyone from the United Nations to ordinary people are concerned about at present, and its advantages and disadvantages, trade-offs, evolution, and development are the motivation and nutrition source for us to think about related academic issues.

The fourth is the field of linguistics. The generation and development of language is also an important area that Holland once paid attention to. As can be seen from Holland's biography, he once cooperated with scholars in Hong Kong to explore the rich phenomena and the inherent laws in the formation and history of multiple dialects in some parts of Fujian. For example, how villages not far away can have very different dialects for a long time. In this regard, our attention and research are far from enough. It is important to note that Holland, who devoted himself to this kind of research, was not motivated by curiosity, but was convinced that there was a law behind these colorful phenomena that we have not yet grasped. In my opinion, this is a research direction that is very necessary to attract more attention from Chinese and foreign scholars.

The fifth is the field of history and sociology. Recently, I read a book that is very influential in the social field: *The Imaginary Community* [14]. The author creatively combs and studies the history of the formation and evolution of the nation and puts forward quite novel views. Despite the ideological controversy, in terms of research methods, the author does give us much inspiration. The author argues convincingly that the formation of many nations is precisely in the process of mutual games and trade-offs between groups of different backgrounds. In this process, production methods, material products and climate, and even some "small things" such as demography, mapping, museum establishment, etc., all played an unexpected key role in this evolution. I believe that the research in this area will be very interesting and meaningful, and it will also be a strong support for the study of the game phenomenon of heterogeneous agents.

6. Closing Remarks

This paper is not an academic paper in the strict sense, but the result of a preliminary combing based on years of confusion and thinking. The purpose is to share some sparks of thinking with colleagues, so as to further study and develop this very interesting and wonderful field.

As stated earlier, this is not a paper in a strict sense, but some exploratory thinking notes. I take the liberty of putting forward some views and the purpose is nothing more than to enter into discussions with interested colleagues in the journey of scientific exploration.

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